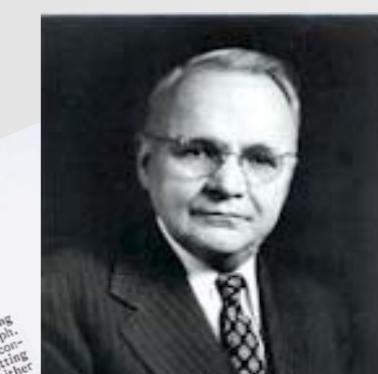


History of the Sampling Theorem

Sampling Theory

Distortionless transmission of telegraphic (digital) signals

Nyquist, H., "Certain factors affecting telegraph speed", *Bell Syst. Tech. J.*, Vol. 3, Apr. 1924, 324-346.



Interpolation of sampling pulses of analog signals

Band-limited signals



1908

Whittaker, E.T., "On the functions expansions of the interpolation theory", *Proc. Roy. Soc. Edinburgh*, Vol. 35, 1915, 181-194.



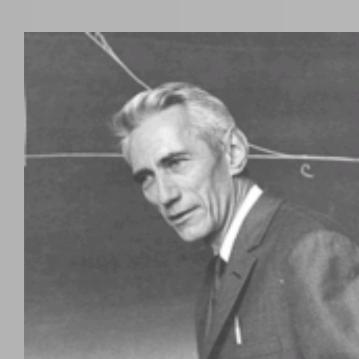
1919

Ogura, K., "On a certain transcendental integral function in the theory of interpolation", *Tohoku Math. J.*, 17, 1920, 64-72.



1927

Kotelnikov, V.A., "On the carrying capacity of the "ether" and wire in telecommunications", *Material for the First All-Union Conference on Questions of Communications*, Izd. Red.Upr. Svyazi RKKA, Moscow, 1933.

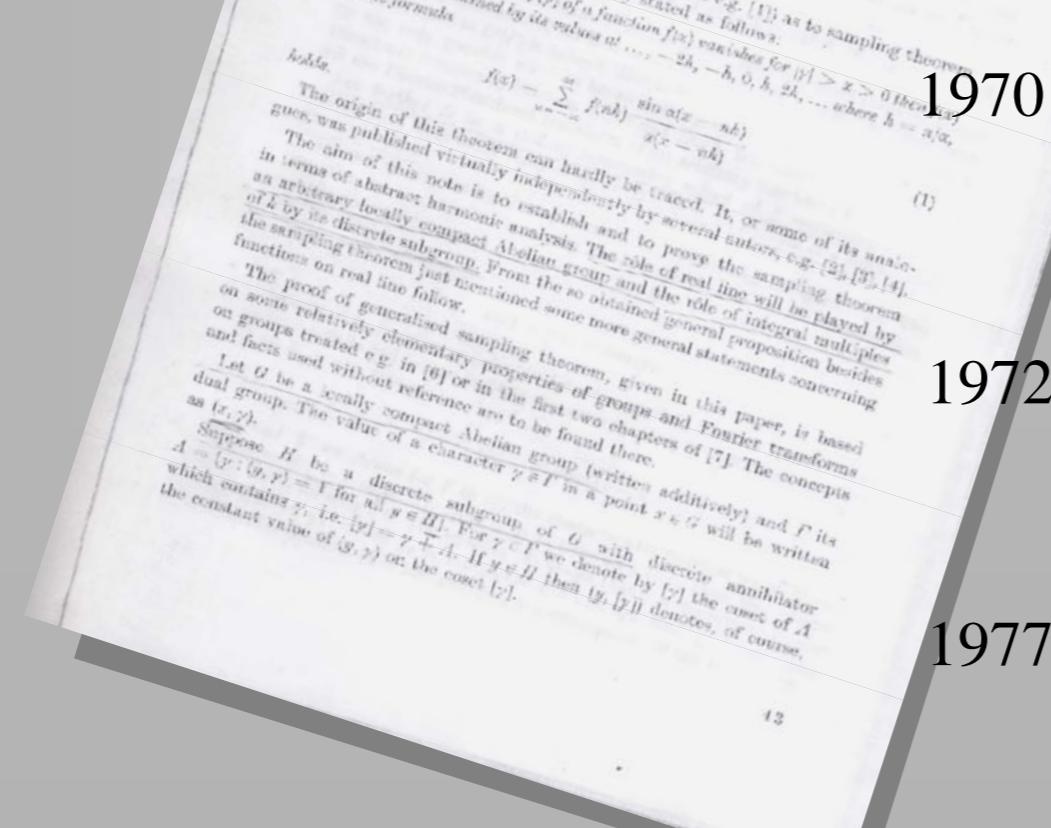


Raabe, H., "Untersuchungen an der wechselzeitigen Mehrfachübertragung (Multiplexübertragung)", *Elektrische Nachrichtentechnik*, Vol. 16, 1939, 213-228.



Shannon, C.E., "A mathematical theory of communication", *Bell System Tech. J.*, Vol. 27, 1948, 379-423.

Someya, I., *Hakei Denso (Waveform Transmission)*, Shikyo, Tokyo, 1949.



1965

In the literature on information theory (see e.g. [1]) as to sampling theorem is referred to the assertion roughly stated as follows: "If the Fourier transform $\hat{f}(\omega)$ exists for all $\omega \geq 0$ then $f(x) = \sum_{n=-\infty}^{\infty} f(n) e^{inx}$ for all x if and only if $\hat{f}(\omega) = 0$ for all $\omega > \pi$ ". The origin of this theorem can hardly be traced. It is one of its unique features that it is published virtually independently by several authors, e.g. [2], [3], [4]. The aim of this note is to establish and to prove the sampling theorem in terms of abstract harmonic analysis. The role of the real line will be played by an arbitrary locally compact Abelian group and the role of integer multiples of \mathbb{Z} by its discrete subgroup. From the more general statements concerning the sampling theorem, first mentioned some more general statements concerning the sampling theorem follow.

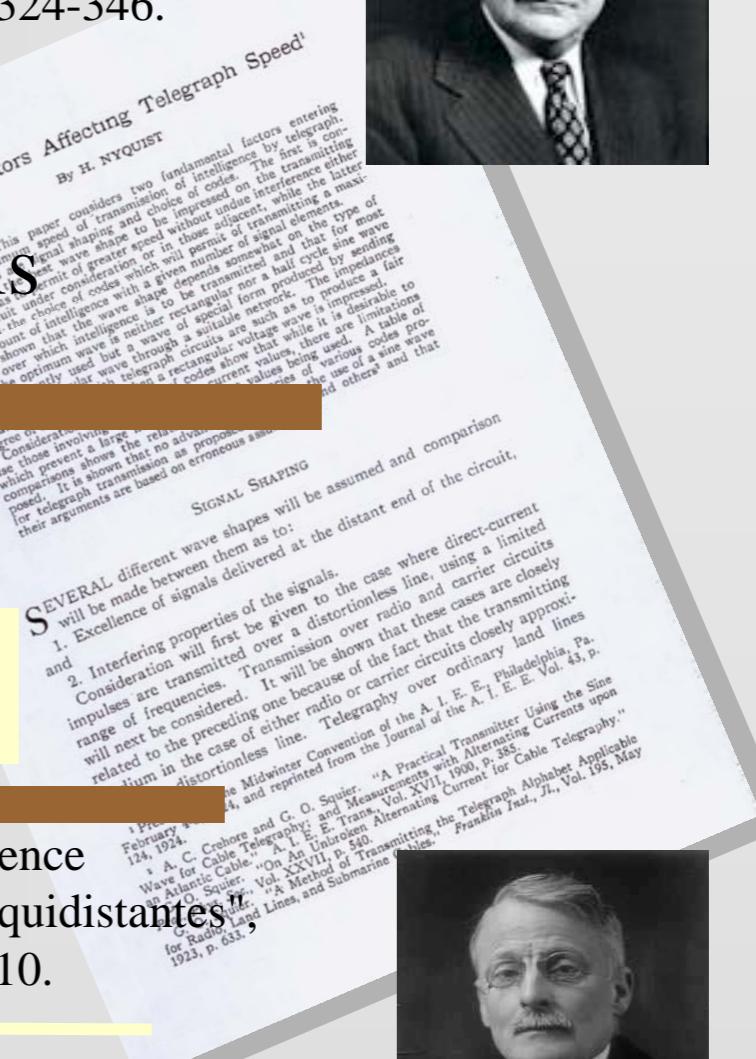
The proof of generalized sampling theorem, given in this paper, is based on some relatively elementary properties of groups and Fourier transforms on groups treated e.g. in [6] or in the first two chapters of [7]. The concepts and facts used without reference are to be found there.

Let G be a locally compact Abelian group with identity e . Let Γ be a discrete subgroup of G with discrete annihilator Γ^\perp . Let χ_Γ be a character of Γ and χ_Γ^\perp be a character of Γ^\perp . Then χ_Γ^\perp is a character of G and $\chi_\Gamma^\perp(e) = 1$.

Suppose H be a discrete subgroup of G with discrete annihilator H^\perp which contains Γ , i.e. $\Gamma^\perp \subseteq H^\perp$. If χ_Γ^\perp denotes the constant value of χ_Γ^\perp on H^\perp , then $\chi_\Gamma^\perp|_{H^\perp}$ denotes, of course, the constant value of χ_Γ^\perp on the coset H^\perp .

Duration-limited signals

De la Vallée Poussin, Ch.-J., "Sur la convergence des formulae d'interpolation entre ordonnées équidistantes", *Bull. Cl. Sci. Acad. Roy. Belg.*, 4, 1908, 319-410.



1915

Theis, M., Über eine Interpolations formel von de la Vallée Poussin", *Math. Z.*, 3, 1919, 93-113.

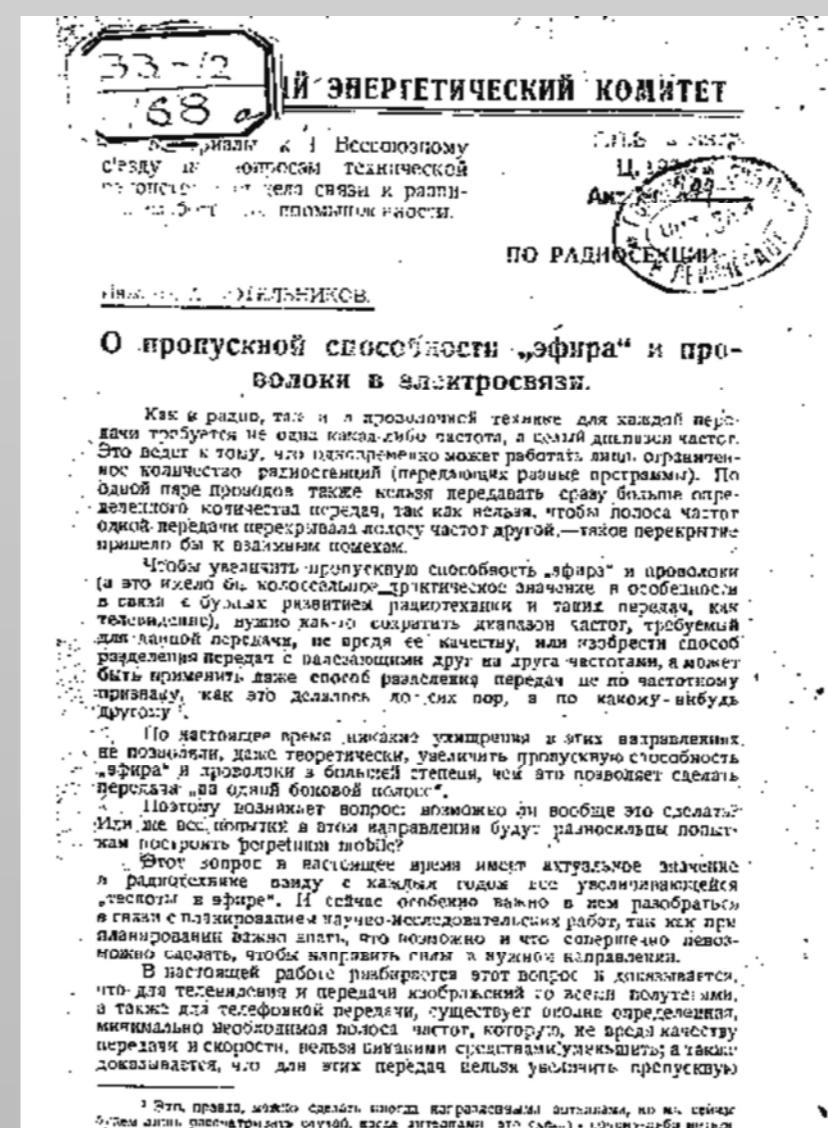


1920

Whittaker, J.M., "The "Fourier" theory of the cardinal function", *Proc. Edinburgh Math. Soc.*, 2, 1927-1929, 169-176.



1933



1939

1948

1949

I. Kluvanek

Kluvánek, I., "Sampling theorem in abstract harmonic analysis", *Mat. Fiz. Časopis Slovens. Akad.*, Vied. 15, 1965, 43-48.

F. Pichler

Pichler, F.R., "Sampling theorem with respect to Walsh-Fourier analaysis", Appendix B in *Reports Walsh Functions and Linear System Theory*, Elec. Eng., Dept., Univ. of Maryland, College Park, May 1970.

C.T. Le Dinh, P. Le, R. Goulet

Le Dinh, C. T., Le, P., Goulet, R., "Sampling expansions in discrete and finite Walsh-Fourier analysis", *Proc. 1972 Symp. Applic. Walsh Functions*, Washington, D.C., USA, 265-271.

P.L. Butzer, W. Splettstosser

Butzer, P.L., Splettstosser, W., "Sampling principle for Duration limited signals and dyadic Walsh analysis", *Information Science*, Vol. 14, 1978, 93-106.

