CASCADE REALIZATION OF 3-INPUT 3-OUTPUT CONSERVATIVE LOGIC CIRCUITS
----- Application of Three-valued Logic to Two-valued Logic -----

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A conservative logic element (CLE) is a multiple-output logic element whose weight of an input vector is equal to that of the corresponding output vector, and is a generalized model of magnetic bubble logic elements ,etc. Arbitrary 3-input 3-output conservative logic circuits (3-3 CLC's) are realized by cascade connections of 3-input 3-output CLE's called "primitives". It is shown that the necessary and sufficient number of different primitives to realize an arbitrary 3-3 CLC is three in the case when the crossovers of lines are permitted. The method of realizations is similar to that of three-valued one-variable two-output logic functions.

Three-valued Representation of 3-3 CLC

Vectors of three-variable can be classified into four classes according to their weights. We assign a number to each vector $\underset{\sim}{a}=(a_1,a_2,a_3)$ as Table 1.

Table 1

weight	$a_1a_2a_3$	number
0	0 0 0	1
1	1 0 0	1
	0 1 0	2
	0 0 1	3
2	0 1 1	1
	1 0 1	2
	1 1 0	3
3	1 1 1	1

Table 2

weight	input	output
0	1	1
1	1	a_1
	2	a_2
	3	a_3
2	1	b_1
	2	b_2
	3	b_3
3	1	1

$a_1,a_2,a_3,b_1,b_2,b_3\in\{1,2,3\}$

By the definition of CLC, the weight of an input vector is equal to that of the corresponding output vector, so any 3-3 CLC can be represented as Table 2. For the inputs of weight 0 and 3, the outputs are uniquely determined. So instead of two-valued representation, it can be represented as (A). (A) is called three-valued representation of 3-3 CLC. Any 3-3 CLC which is represented as (A) can be regarded as a three-valued one-variable two-output logic function of Table 3.

$$\mu=\frac{(a_1\ a_2\ a_3)}{(b_1\ b_2\ b_3)} \quad \text{--- (A)}$$

Table 3

input	1	2	3
output 1	a_1	a_2	a_3
output 2	b_1	b_2	b_3

Universal Set of Primitives

Lemma 1: If two 3-3 CLC's μ_1 and μ_2 are connected in a cascade, then 3-3 CLC $\mu_1\mu_2$ is realized, where

$$\mu_1=\frac{(a_1\ a_2\ a_3)}{(b_1\ b_2\ b_3)},\quad \mu_2=\frac{(c_1\ c_2\ c_3)}{(d_1\ d_2\ d_3)},\ \text{and}$$

$$\mu_1\mu_2=\frac{(a_1\ a_2\ a_3)\cdot(c_1\ c_2\ c_3)}{(b_1\ b_2\ b_3)\cdot(d_1\ d_2\ d_3)}\ .$$

"$\cdot$" denotes the composition of transformations and $(a_1\ a_2\ a_3)\cdot(c_1\ c_2\ c_3)=(c_{a_1}\ c_{a_2}\ c_{a_3})$.

The set of all 3-3 CLC's is denoted by K.

Definition 1: Let $M=\{\mu_1,\mu_2,\dots,\mu_p\}$ be a subset of K. The minimal set S which satisfies the following conditions is said to be the set of composed functions, written [M] of $[\mu_1,\mu_2,\dots,\mu_p]$:(i) $M\subset S$,(ii) $\mu_i,\mu_j\in S \Rightarrow \mu_i\cdot\mu_j,\ \mu_j\cdot\mu_i\in S$.

$[\mu_1,\mu_2,\dots,\mu_p]$ represents the set of the circuits which are obtained by cascade connections of $\mu_1,\mu_2,\dots,\mu_p$. It is clear that $[M]\subset K$ by definition. If $[M]=K$, then M is said to be a universal set of 3-3 CLC's.

Definition 2:
$$\phi_1(\mu)=\delta(a_1-a_2)+\delta(a_2-a_3)+\delta(a_3-a_1)$$
$$\phi_2(\mu)=\delta(b_1-b_2)+\delta(b_2-b_3)+\delta(b_3-b_1)$$
$$\psi(\mu)=\delta(a_1-b_1)+\delta(a_2-b_2)+\delta(a_3-b_3)$$

where, $\mu=\dfrac{(a_1\ a_2\ a_3)}{(b_1\ b_2\ b_3)}\in K$ and $\delta(x)=\begin{cases}0 & \text{if } x\neq 0\\ 1 & \text{if } x=0\end{cases}$.

Lemma 2: Let $\mu_1,\ \mu_2\in K$. For i=1,2

(1) $\phi_i(\mu_1)=\phi_i(\mu_2)=0 \Rightarrow \phi_i(\mu_1\cdot\mu_2)=0$

(2) $\phi_i(\mu_1)=3 \Rightarrow \phi_i(\mu_1\cdot\mu_2)=\phi_i(\mu_2\cdot\mu_1)=3$

(3) $\phi_i(\mu_1)\neq 0 \Rightarrow \phi_i(\mu_1\cdot\mu_2)\neq 0,\ \phi_i(\mu_2\cdot\mu_1)\neq 0$

(4) $\phi_i(\mu_1)=0,\ \phi_i(\mu_2)=1 \Rightarrow \phi_i(\mu_1\cdot\mu_2)=\phi_i(\mu_2\cdot\mu_1)=1$

(5) $\psi(\mu_1)=\psi(\mu_2)=3 \Rightarrow \psi(\mu_1\cdot\mu_2)=\psi(\mu_2\cdot\mu_1)=3$

Definition 3:
$M_i=\{\mu\mid\mu\in K,\ \phi_i(\mu)=0\}$ (i=1,2)
$L_i=\{\mu\mid\mu\in K,\ \phi_i(\mu)=3\}$ (i=1,2)
$N_i=\{\mu\mid\mu\in K\ \ \psi(\mu)=i\}$ (i=0,1,2,3)

Lemma 3: $[M_i]=M_i$, $[\bar{M}_i]=\bar{M}_i$, $[L_i]=L_i$ (i=1,2), $[N_3]=N_3$, $[M_1\cup\bar{M}_2\cup L_1]=M_1\cup\bar{M}_2\cup L_1$, $[\bar{M}_1\cup M_2\cup L_2]=\bar{M}_1\cup M_2\cup L_2$, $[\bar{M}_1\cup\bar{M}_2]=\bar{M}_1\cup\bar{M}_2$, $[(N_0\cup N_3)\cap M_1\cap M_2]=(N_0\cup N_3)\cap M_1\cap M_2$.

Theorem 1: If M is universal, then M contains three different elements μ_1, μ_2, and μ_3 such that $\mu_1\in\bar{M}_1\cap M_2\cap\bar{L}_1$, $\mu_2\in M_1\cap\bar{M}_2\cap\bar{L}_2$, and $\mu_3\in M_1\cap M_2\cap N_1$.

Lemma 4: Any three-valued one-variable logic function $T=(t_1\ t_2\ t_3)$ can be realized by the three diffenent primitive functions P=(2 1 3), C=(2 3 1), and D=(1 1 3).

Lemma 5: Any 3-3 CLC can be realized as a cascade connections of six different primitives $\frac{P}{I}$, $\frac{C}{I}$, $\frac{D}{I}$, $\frac{I}{P}$, $\frac{I}{C}$, and $\frac{I}{D}$, where I=(1 2 3).

Note that these six primitives are composed of only three primitives when the crossovers of lines are permitted.

Lemma 6: Any 3-3 CLC can be realized as a cascade connection of three different primitives

$$\frac{(2\ 2\ 3)}{(1\ 2\ 3)},\ \frac{(1\ 2\ 3)}{(2\ 2\ 3)},\ \text{and}\ \frac{(2\ 3\ 1)}{(1\ 3\ 2)}\ .$$

Theorem 2: Any 3-3 CLC can be realized as a cascade connections of three different primitives $\{\mu_1,\mu_2,\mu_3\}$ where $\mu_1\in\bar{M}_1\cap M_2\cap\bar{L}_1$, $\mu_2\in M_1\cap\bar{M}_2\cap\bar{L}_2$, and $\mu_3\in M_1\cap M_2\cap N_1$.

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Reference: R.C.Minnick, P.T.Bailey, R.M.Sandfort and W.L.Semon,"Magnetic bubble logic", 1972 WESCON.