obvious that X_2 should be preferred to X_3 since the X_2 result, whether positive or negative, will affect the class probabilities while X_3 cannot cause any change. In fact, assume that X_2 is tested and a positive result is obtained, then the class probabilities are transformed to

Class	c_1	c_2	c_3	c_4
$\pi^{(3)}$	0.089	0.727	0.104	0.080

The probability of error is still 0.2737. Parenthetically, let us note that if the Shannon information gain rule is used, feature X_2 will indeed be preferred, since its information gain is 0.038 versus information gain of 0 from X_3.

V. Applications

The concept of irrelevant features is of particular importance for problems where experts knowledge about the class structure is widely used. For this type of problems the following approach for model building and utilization is suggested.

Stage 1: Define the classes for the system under consideration. For each of these classes identify the features which are significant for the recognition of this class and estimate, either subjectively or by data, the corresponding conditional distribution. Store these data in List I.

Stage 2: Based on the information contained in List I, generate for each *feature* a list of the classes for which it is relevant and their corresponding probabilities. Call this list List II. For each single feature X_j, review this list and verify that all the relevant classes to this feature are included. In case that a relevant class, say c_i, is missing, revise the pattern of c_i in List I by adding X_j with its conditional probability to the pattern of c_i. Obviously List II is updated accordingly. This review may also suggest modification of subjective probabilities to obtain a proper proportion for the distribution of this feature over the relevant classes.

This approach is being used in the development of a computer-aided medical decision system for emergency and critical care settings, and has been found to be very useful for improving subjectively characterized patterns of medical disorders.

This approach also provides us with an efficient computational tool. Once a feature is observed, its location in List II is addressed. The prior probabilities for its relevant classes are normalized, and the posterior probabilities are computed following (8) and (9) in Theorem 2. This way, instead of storing the whole pattern matrix we only store List II. The saving in storage is substantial when many irrelevance relationships exist. For instance, in a system for diagnosing infertility disorders in females (Schild, Gavish and Lunenfeld [5]; Ben-Bassat [3]), 68 classes (disorders) are characterized by a total of 188 features. However, for most of the classes in this system only 20 to 30 features are relevant, which implies a saving of 60–90 percent by using List II instead of the pattern matrix.

VI. Summary

The concept of irrelevant features in Bayesian models for pattern recognition is introduced, and its mathematical meaning is explained. A technique for computing the conditional probabilities of irrelevant features, if necessary, is described. The effect of irrelevant features on feature selection in sequential classification is discussed and illustrated.

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Realization of Minimum Circuits with Two-Input Conservative Logic Elements

TSUTOMU SASAO and KOZO KINOSHITA

***Abstract*—This correspondence is concerned with the realization of logical functions by using two-input three-output conservative logic elements (CLE's) called I_B.**

A conservative logic element is a multiple-output logic element whose number of "1's" of the input is equal to that of the corresponding output, and whose fan out of each output terminal is restricted to one. In order to realize arbitrary functions, it is necessary to use constant-supplying elements C_1's. The minimum circuit is a circuit which consists of minimum number of C_1's and minimum number of I_B elements. In this correspondence, we give lower bounds on the number of I_B elements in the circuit and two minimum decomposition theorems. These results are useful for the verification of the minimality of a given circuit and for the realization of minimum circuits. Several examples illustrate this.

***Index Terms*—Conservative logic elements (CLE's), logic design, logic minimization, magnetic bubble logic, switching theory.**

I. Introduction

A conservative logic element (CLE) is a multiple-output logic element whose weight of an input vector is equal to that of the corresponding output vector [1]–[3]. In [4], we have shown that not only magnetic bubble logic elements, but also fluid logic elements, transfer relays, current-mode logic elements without power sources, and so on are all CLE's. In order to realize an arbitrary function, it is necessary to use constant-supplying elements (CSE's). A C_1 supplies a constant "1" and a C_0 supplies a constant "0." Both a C_1 and a C_0 are called CSE's. It is assumed that the fanout of each output of a CLE or a CSE is one. In the case of magnetic bubble logic elements, the C_1 corresponds to a magnetic bubble generator and the C_0 corresponds to connecting nothing. In the case of current-mode logic element, the C_1 corresponds to a constant current source and the C_0 corresponds to connecting nothing.

In [8], we have considered the universality of two-input CLE in relation to the number of C_1's. The I_B element shown in Fig. 1 is a two-input three-output CLE and it has been shown that any logic

Manuscript received May 28, 1976; revised May 28, 1977.
The authors are with the Department of Electronic Engineering, Osaka University, Osaka, Japan.

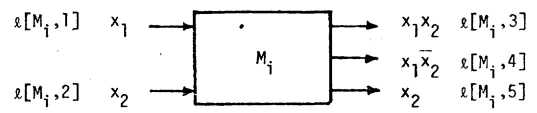

Fig. 1. I_B element.

function can be realized by I_B elements and at most two C_1's. A minimum circuit is a circuit which consists of minimum number of C_1's and minimum number of I_B elements.

In [5], [6], and [8], we have shown that the minimum circuit of most function have the characteristic circuit structure called "1-4 form." Examples of the 1-4 form are shown in Figs. 2, 7, 8, and 10.

In this correspondence, we derive some properties of minimum circuits. These results are useful for the realization of minimum circuits. In Section II, we give some definitions and summarize previous works. In Section III, first we give lower bounds on the number of elements in minimum circuits. These bounds are useful for the verification of the minimality of a given circuit. Second, we give two minimum decomposition theorems. These theorems show that a minimum circuit can be obtained by realizing a minimum circuit of fewer variables in some cases.

II. Definitions and Basic Properties

In this section, we give some definitions and summarize results of previous work [8]. $f(X)$ denotes n-variable function $f(x_1, x_2, \cdots, x_n)$.

Definition 2.1:

1) A function $f(X)$ is said to be the g_{11} function if $f(0, 0, \cdots, 0) = 1$. A set of g_{11} functions is denoted by G_{11}.

2) A function $f(X)$ is said to be the g_2 function if $f \notin G_{11}$ and there exist no x_i such that $f(X) = x_i \cdot g(X\backslash i)$, where $g(X\backslash i)$ represents a function of a set of variables X except x_i. A set of g_2 functions is denoted by G_2.

3) A function $f(X)$ is said to be the g_{12} function if there exists an x_i such that $f(X) = x_i \cdot g(X\backslash i)$ and $g(X\backslash i) \in G_2$. A set of g_{12} functions is denoted by G_{12}.

4) A function $f(X)$ is said to be the g_{01} function if there exists an x_i such that $f(X) = x_i \cdot g(X\backslash i)$and $g(X\backslash i) \in G_{11}$. A set of g_{01} functions is denoted by G_{01}.

5) A function $f(X)$ is said to be the g_{02} function if there exist x_i and x_j such that $f(X) = x_i \cdot x_j \cdot g(X\backslash i, j)$ or if $f(X) \equiv 0$. A set of g_{02} functions is denoted by G_{02}.

Any function belongs to exactly one of the above five sets (see Fig. 3).

Definition 2.2: A set of circuits which consist of I_B elements and k or less than k C_1's is denoted by $\{I_B:k\}$. A set of functions which can be realized by the circuit in $\{I_B:k\}$ is denoted by $[I_B:k]$.

The next theorem shows the necessary and sufficient number of C_1's to realize a given function.

Theorem 2.1: If $f \in G_{01} \cup G_{02}$ then $f \in [I_B:0]$. If $f \in G_{11} \cup G_{12}$ then $f \in [I_B:1]$ and $f \notin [I_B:0]$. If $f \in G_2$ then $f \in [I_B:2]$ and $f \notin [I_B:1]$ and $f \notin [I_B:0]$.

Definition 2.3: A circuit which realizes a function f with m I_B elements and minimum number of C_1's is denoted by $R_m(f)$. $R_m(f)$ is said to be minimum if there is no $R_s(f)$ such that $s < m$.

Definition 2.4: For a I_B element, the input lines and output lines are denoted as shown in Fig. 1. $l[M_i, 1]$ is said to be the immediate predecessor of $l[M_i, 3]$ and $l[M_i, 4]$. $l[M_i, 2]$ is said to be the immediate predecessor of $l[M_i, 5]$. The function corresponding to line l_i is denoted by $f(l_i)$. For simplicity, $f(l[M_i, j])$ is denoted by $f(M_i, j)$.

Definition 2.5: Let P: $l_0, l_1, \cdots, l_m$ be a sequence of lines in a circuit. P is called a path if l_{i-1} is the immediate predecessor of l_i for $i = 1, 2, \cdots, m - 1$. Especially when l_0 is an input line and l_m is an output line of the circuit, P is called the I-O path. A path P is said to have 1-4 form if all lines $l[M_i, 1]$ and $l[M_i, 4]$ are included in the path for all elements M_i which are connected to the lines in P. The circuit is said to have the 1-4 form if the I-O path has the 1-4 form.

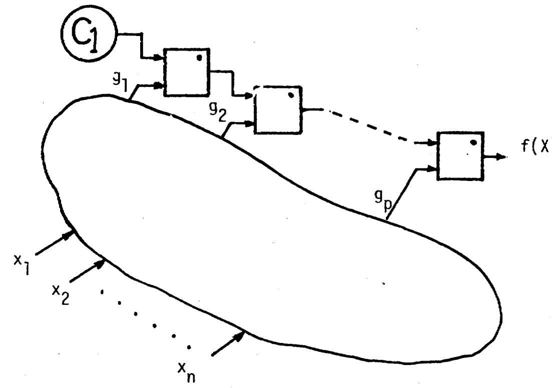

Fig. 2. A typical example of the 1-4 form.

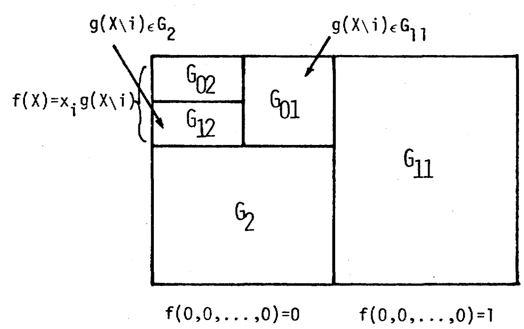

Fig. 3. Classification of functions.

Lemma 2.1: Let $l_0, l_1, \cdots, l_p$ be a path of a circuit. Then

$$f(l_0) \supseteq f(l_1) \supseteq \cdots \supseteq f(l_p).$$

The next theorem shows that minimum circuit of most functions have the characteristic circuit structure called 1-4 form.

Theorem 2.2: Let $R_m(f)$ be a minimum circuit. $R_m(f)$ has the 1-4 form if and only if $f \notin G_{02}$.

Definition 2.6: Either the variable or the C_1 which is connected to the input line of a path is said to be the source of the path.

By Theorems 2.1 and 2.2, and Lemma 2.1, the following theorem is obtained.

Theorem 2.3: In a minimum circuit $R_m(f)$, the source of the I-O path is C_1 if $f \in G_{11} \cup G_2$, or x_i if $f \in G_{01} \cup G_{12}$ and $(X) = x_i \cdot g(X\backslash i)$.

Lemma 2.2: Let $R_m(g(X\backslash i))$ be a minimum circuit and $g(X\backslash i) \in G_{11} \cup G_2$. If the source of the I-O path is replaced by a variable x_i, then a minimum circuit $R'_m(f)$ which realizes the function $f(X) = x_i \cdot g(X\backslash i)$ is obtained.

Proof: Similar to Lemma 5.6 of [8]. Q.E.D.

Theorem 2.3 and Lemma 2.2 imply that if $f(X) \in G_{01} \cup G_{12}$ and $f(X) = x_i g(X\backslash i)$, then a minimum circuit of $f(X)$ can be obtained by the minimum circuit of $g(X\backslash i) \in G_{11} \cup G_2$.

III. Realization of Minimum Circuits

In this section, first we give some lower bounds on the number of elements in a circuit. These bounds are useful for the verification for the minimality of a given circuit.

The number of elements in the I-O path of $R_m(f)$ is denoted by $L(R_m(f))$.

Lemma 3.1: In a minimum circuit $R_m(f)$, if $\bar{f} \in G_2$ then $L(R_m(f)) \geq 2$.

Proof: $\bar{f} \in G_2$ implies $f \in G_{11}$. Since the source of the I-O path is C_1 and $R_m(f)$ has the 1-4 form, if $L(R_m(f)) = 1$ then the circuit must be in the form shown in Fig. 4. But to realize $\bar{f} \in G_2$, it requires two C_1's. This contradicts the condition of $R_m(f)$. Q.E.D.

Lemma 3.2: In a minimum circuit $R_m(f)$, if $f \in G_{11}$ then $L(R_m(f)) \geq k$, where

$$k = \sum_{i=1}^{n} \overline{f(e_i)} \text{ and } e_i = (0, 0, \cdots, 0, \underset{i}{1}, 0, \cdots, 0).$$

Proof: Since the source of the I-O path is C_1 and $R_m(f)$ has 1-4 form, f can be represented as $f = \bar{g}_1 \cdot \bar{g}_2 \cdots \bar{g}_p$ or $\bar{f} = g_1 \vee g_2 \vee \cdots \vee g_p$, where $g_1, g_2, \cdots, g_p$ correspond to the functions shown in Fig. 2. When $\overline{f(e_i)} = 1$, at least one g_t in $\{g_1, g_2, \cdots, g_p\}$ becomes to "1." $g_t \notin G_{11} \cup G_2$, because if not so it contradicts the definition of $R_m(f)$. So g_t can be written in a form $g_t = x_i h(X\backslash i)$ such that $h(X\backslash i) \in G_{11}$. If $\overline{f(e_i)} = \overline{f(e_j)} = 1$ $(i \neq j)$, then there exist two functions such that $g_{t_1} = x_i h_1(X\backslash i)$ and $g_{t_2} = x_j h_2(X\backslash j)$ in $\{g_1, g_2, \cdots, g_p\}$. Therefore $p \geq k$. Q.E.D.

Lemma 3.3: In a minimum circuit $R_m(f)$, if $f \in G_{11}$ satisfies the conditions 1) and 2), then $L(R_m(f)) \geq {}_nC_2$.

1) $f = 1$ when exactly one input is "1."

2) $f = 0$ when exactly two inputs are "1."

Proof: Since $f \in G_{11}$, $R_m(f)$ has the 1-4 form and the source of the I-O path is C_1. Similar to Lemma 3.2, $\bar{f}$ is represented as $\bar{f} = g_1 \vee g_2 \vee \cdots \vee g_p$. Conditions 1) and 2) imply that some of g_k's can be written as $g_{ij} = x_i x_j h(X\backslash i, j)$ such that $h(X\backslash i, j) \in G_{11}$. Condition 2) also implies that $\{g_1, g_2, \cdots, g_p\}$ includes all g_{ij} for all pairs of input variables. Therefore $p \geq {}_nC_2$. Q.E.D.

Lemma 3.4: In a minimum circuit $R_m(f)$, if $f \notin G_{02}$ then $l[M_i, 2]$ of the element M_i in the I-O path is not connected to $l[M_j, 5]$ of other elements M_j.

Proof: Suppose that $l[M_j, 5]$ is connected to $l[M_i, 2]$. As $f \notin G_{02}$, $R_m(f)$ has the 1-4 form and $l[M_i, 4]$ is in the I-O path. $f(M_j, 2) = 0$ implies $f(M_j, 3) = 0$, and $f(M_j, 2) = 1$ implies $f(M_i, 4) = 0$ and $f(X) = 0$ independently of values of $f(M_j, 3)$ and $f(M_j, 4)$. So M_j can be replaced by two lines as shown in Fig. 5(b) without changing the output function. This contradicts that $R_m(f)$ is minimum. Q.E.D.

Lemma 3.5: Let $R_m(f)$ be a minimum circuit and $f \in G_{11}$. If f cannot be written in a form $f = \bar{x}_i g(X\backslash i)$ then $m \geq 2p$, where $p = L(R_m(f))$.

Proof: Let M_i be an element in the I-O path. By Lemma 3.4, $l[M_i, 2]$ is connected to either $l[M_j, 3]$ or $l[M_j, 4]$ or a variable. Since $R_m(f)$ has the 1-4 form, if both $l[M_j, 3]$ and $l[M_j, 4]$ are connected to the elements in the I-O path, then it can be represented as Fig. 6. By Lemma 2.1, $f(X)$ can be written as $f(X) = \overline{f(M_j, 3)} \cdot \overline{f(M_j, 4)} h$. Note that $\overline{f(M_j, 3)} \cdot \overline{f(M_j, 4)} = \overline{f(M_j, 1)}$. M_j can be replaced by two lines shown in Fig. 5(b) without changing the output function. Therefore, the second terminal of each element in the I-O path is connected to a different element or a variable. If $l[M_i, 2]$ is connected to a variable x_j, then $f(X)$ can be written in a form $f(X) = \bar{x}_j h(X\backslash j)$ and this contradicts the assumption. Therefore $m \geq 2p$. Q.E.D.

Example 3.1: Let $R_m(f)$ be a minimum circuit of $f = \bar{x}_1 \bar{x}_2 \bar{x}_3 \vee x_2 x_3$. As $f \in G_{11}$ and $\overline{f(e_1)} + \overline{f(e_2)} + \overline{f(e_3)} = 3$, $L(R_m(f)) \geq 3$ by Lemma 3.2. As $f(X)$ cannot be written in a form $f(X) = \bar{x}_i \cdot g(X\backslash i)$, $m \geq 6$ by Lemma 3.5. $f(X)$ can be realized as Fig. 7 with 6 elements, so it is minimum.

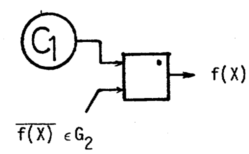

Fig. 4. Proof of Lemma 3.1.

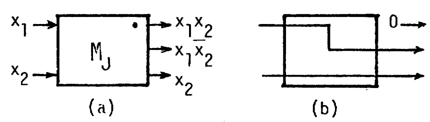

Fig. 5. Redundant element.

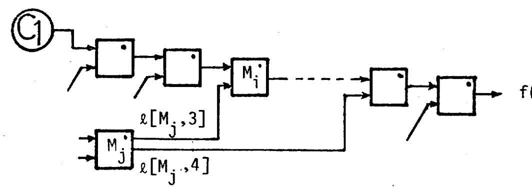

Fig. 6. Proof of Lemma 3.5.

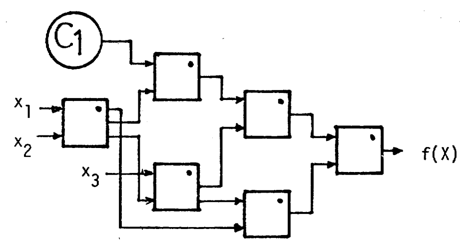

Fig. 7. Realization of $f = \bar{x}_1 \cdot \bar{x}_2 \cdot \bar{x}_3 \vee x_2 \cdot x_3$.

Example 3.2: Let $R_m(f)$ be a minimum circuit of $f = \overline{x_1 \cdot x_2} \cdot \overline{x_1 \cdot x_3} \cdot \overline{x_1 \cdot x_4} \cdot \overline{x_2 \cdot x_3} \cdot \overline{x_2 \cdot x_4} \cdot \overline{x_3 \cdot x_4}$. As f satisfies the conditions of Lemma 3.3, $L(R_m(f)) \geq 6$. As $f(X)$ cannot be written in a form $f(X) = \bar{x}_i \cdot g(X\backslash j)$, $m \geq 12$ by Lemma 3.5. Therefore, the circuit shown in Fig. 8 is minimum.

These lemmas stated above give lower bounds on $L(R_m(f))$ such that $f \in G_{11}$. The next lemma gives an upper bound on $L(R_m(f))$ such that $f \in G_{11} \cup G_2$.

Lemma 3.6: In a minimum circuit $R_m(f)$, if $f \in G_{11} \cup G_2$ then

$$L(R_m(f)) \leq \sum_{a_i \in B^n} \overline{f(a_i)}, \quad \text{where } B = \{0, 1\}.$$

Proof: As $f \in G_{11} \cup G_2$, $R_m(f)$ has the 1-4 form. Similar to Lemma 3.2, $\bar{f}$ can be written as $\bar{f} = g_1 \vee g_2 \vee \cdots \vee g_p$. Let the minterm expansion of $\bar{f}$ be $\bar{f} = m_{i_1} \vee m_{i_2} \vee \cdots \vee m_{i_k}$. For any $g_s \in \{g_1, g_2, \cdots, g_p\}$, there exists m_{i_j} such that $m_{i_j} \subset g_s$ and $m_{i_j} \not\subset (g_1 \vee g_2 \vee \cdots \vee g_{s-1} \vee g_{s+1} \vee \cdots \vee g_p)$. Otherwise, $(g_1 \vee g_2 \vee \cdots \vee g_{s-1} \vee g_{s+1} \vee \cdots \vee g_p) \supset g_s$, which means that the element of the I-O path whose line is connected to g_s can be omitted. It is clear that different g_s's correspond to different m_{i_j}'s. Therefore $k \geq p$. Q.E.D.

Example 3.3: Let $R_m(f)$ be a minimum circuit of $f = \overline{x_1 x_2 \vee x_2 x_3 \vee x_3 x_1 \vee \bar{x}_1 \bar{x}_2 \bar{x}_3}$. As $f \in G_{11}$ and $\overline{f(e_1)} + \overline{f(e_2)} + \overline{f(e_3)} = 3$, $L(R_m(f)) \geq 3$ by Lemma 3.2. On the other hand, $\sum_{a_i \in B^n} \overline{f(a_i)} = 3$, so $L(R_m(f)) \leq 3$ by Lemma 3.6. Therefore, $L(R_m(f)) = 3$.

The next two theorems show that some minimum circuits can be obtained by realizing circuits of fewer variables. These two theorems are called minimum decomposition theorems.

Theorem 3.1: If f can be written in a form $f(X) = g(x_i \bar{x}_j, X\backslash i, j)$, then a minimum circuit for f can be obtained from the minimum circuit of $g(u, X\backslash i, j)$ as shown in Fig. 9.

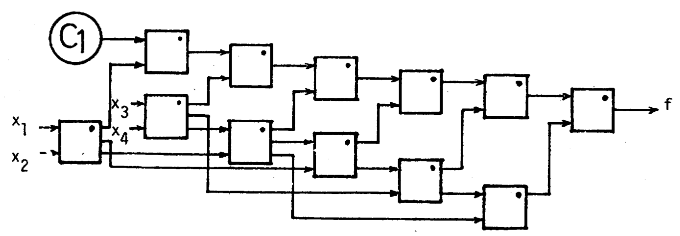

Fig. 8. Realization of $f = \overline{x_1 \cdot x_2} \cdot \overline{x_1 \cdot x_3} \cdot \overline{x_1 \cdot x_4} \cdot \overline{x_2 \cdot x_3} \cdot \overline{x_2 \cdot x_4} \cdot \overline{x_3 \cdot x_4}$.

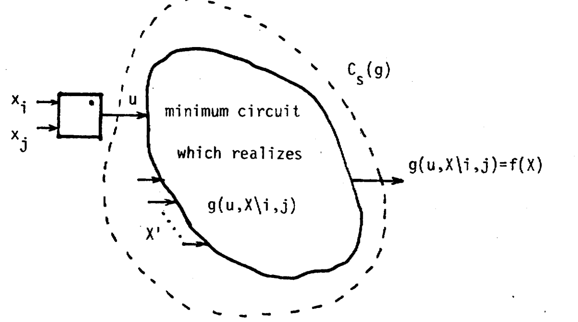

Fig. 9. Realization of $f(X) = g(x_i \bar{x}_j, X\backslash i, j)$.

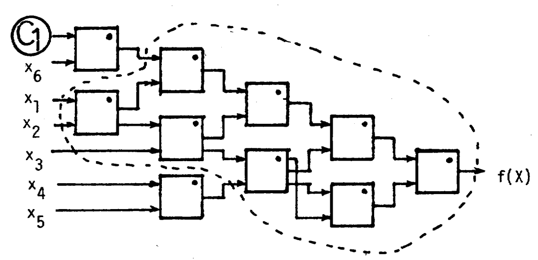

Fig. 10. Realization of $f = \bar{x}_6 \cdot ((\bar{x}_1 \vee x_2)x_3 \cdot x_4 \cdot \bar{x}_5 \vee \bar{x}_1 \cdot \bar{x}_2 \cdot \bar{x}_3 \cdot (\bar{x}_4 \vee x_5))$.

Proof: Let $R_m(f)$ and $C_s(g)$ be minimum circuits. By setting $x_j = 0$ in $R_m(f)$, the element which is connected to x_j can be removed, and the circuit $R'_{m-1}(g)$ which realizes $g(x_i, X\backslash i, j)$ can be obtained. By modifying $C_s(g)$ as shown in Fig. 9, a circuit $C'_{s+1}(f)$ which realizes f can be obtained. Since $R_m(f)$ and $C_s(g)$ are minimum circuits, $s + 1 \geq m$ and $m - 1 \geq s$. Thus $m = s + 1$: Therefore C'_{s+1} is minimum. Q.E.D.

Theorem 3.2: If f can be written in a form $f(X) = \bar{x}_j g(X\backslash j)$ such that $g(X\backslash j) \in G_{11} \cup G_2$, then a minimum circuit can be obtained from the circuit of $g(X\backslash j)$ as shown in Fig. 9 by setting $x_i = 1$.

Proof: Note that f can be written as $f(X) = (1 \cdot \bar{x}_j)g(X\backslash j)$. The proof is similar to Theorem 3.1. Q.E.D.

Example 3.4: A function

$$f = \bar{x}_6 \cdot ((\bar{x}_1 \vee x_2)x_3 \cdot x_4 \cdot \bar{x}_5 \vee \bar{x}_1 \cdot \bar{x}_2 \cdot \bar{x}_3 \cdot (\bar{x}_4 \vee x_5))$$

can be written as $f = \bar{x}_6 \cdot g(x_1, x_2, x_3, u)$, where $g(x_1, x_2, x_3, u) = (\bar{x}_1 \vee x_2) \cdot x_3 \cdot u \vee \bar{x}_1 \cdot \bar{x}_2 \cdot \bar{x}_3 \cdot \bar{u}$ and $u = x_4 \cdot \bar{x}_5$. First, realize a minimum circuit $R_s(g)$: As $g \in G_{11}$ and $\overline{g(e_1)} + \overline{g(e_2)} + \overline{g(e_3)} + \overline{g(e_4)} = 4$, $L(R_s(g)) \geq 4$. As g cannot be written in a form $g = \bar{x}_i \cdot g'(X\backslash i)$, $s \geq 8$. Therefore, the circuit of the inside of the broken line in Fig. 10 is a minimum circuit. Second, realize a minimum circuit of f by using $R_s(g)$. The circuit in Fig. 10 is minimum by Theorems 3.1 and 3.2.

IV. Conclusion

In this correspondence, lower bounds on the number of the I_B elements of a minimum circuit, and two minimum decomposition theorems are obtained. The results of this correspondence are useful for the verification of minimality of a given circuit and for the realization of minimum circuits.

In the case of three-variable functions, a minimum circuit for each function was obtained [5], [7]. It is well known that the 256 logic functions of three-variable can be partitioned into 80 equivalence classes. Two functions are equivalent if and only if one can be obtained from the other by a permutation of the input variables [9]. Only four of the 80 three-variable functions belong to G_{02}: 0, $x_1 \cdot x_2$, $x_1 \cdot x_2 \cdot x_3$, and $x_1 \cdot x_2 \cdot \bar{x}_3$. And minimum circuits for these functions can be easily obtained. Eight of the 80 three-variable functions belong to $G_{01} \cup G_{12}$. By Lemma 2.2, minimum circuits for these functions are obtained by the minimum circuits for corresponding G_{11} or G_2 functions. And the rest of the functions belongs to $G_{11} \cup G_2$. By using results of this paper, the minimality of circuits for 42 of the 80 three-variable functions could be verified. The minimality of the circuits for the other functions were verified by using the assistance of computers.

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