Cascade Realization of 3-Input 3-Output Conservative Logic Circuits

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Abstract—**A conservative logic element (CLE) is a multiple-output logic element whose weight of an input vector is equal to that of the corresponding output vector, and is a generalized model of magnetic bubble logic elements, fluid logic elements, and so on. This paper considers the problem of realizing arbitrary 3-input 3-output conservative logic elements (3-3 CLC's) by cascade connections of 3-input 3-output CLE's called "primitives." It is shown that the necessary and sufficient number of different primitives to realize an arbitrary 3-3 CLC is three in the case when the crossovers of lines are permitted, and four in the case when the crossovers of lines are not permitted.**

***Index Terms*—Cascade realization, conservative logic element (CLE), logic primitives, magnetic bubble logic, minimum circuit, three-valued logic, universality of logic elements.**

I. INTRODUCTION

A CONSERVATIVE logic element (CLE) is a multiple-output logic element whose weight of input vector is equal to that of the corresponding output vector [1], [6]–[8]. In [2], we have shown that not only magnetic bubble logic elements [1], [6]–[11], but also fluid logic elements, transfer relays, current mode logic elements without power sources and so on are all CLE's. In [3] and [4], we have considered the problem of universality of CLE's in relation to the number of constant-supplying elements.

In this paper, we consider the problem of realizing arbitrary 3-input 3-output conservative logic circuits (3-3 CLC's) by cascade connections of 3-input 3-output CLE's (3-3 CLE's) called *primitives*. It is known that there are 729 CLE's [7], and certain of the CLE's are more "desirable" than others. It is useful to study the method for realizing any CLE's by using desirable elements.

R. C. Minnick *et al.* [6] considered the cascade realization of 3-input 3-output magnetic bubble logic circuits: they chose a set of seven most desirable primitives out of 729 3-3 CLC's, and then cascaded these seven primitives in all possible combinations and orientations by using the assistance of a computer.

Here, we consider the necessary and sufficient number of different primitives to realize an arbitrary 3-3 CLC by a cascade connection of primitives.

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II. CONSERVATIVE LOGIC ELEMENTS AND THEIR DUALITY

In this section, we consider the duality of n-n CLE's. This result will be used for the realization of CLC's in Section V.

First, we will define the CLE.

Definition 1: An n-input n-output logic element is said to be a CLE if it satisfies the following condition: for any input vector

$$\boldsymbol{a} = (a_1,a_2, \cdots ,a_n) \in B^n, B = \{0,1\}, \sum_{i=1}^{n} a_i = \sum_{j=1}^{n} y_j(\boldsymbol{a}),$$

where y_j denotes the jth output function.

Definition 2: Let the output function of n-input n-output logic element $\underset{\sim}{A}$ be $y_1,y_2, \cdots ,y_n$. The dual element of $\underset{\sim}{A}$, written $\underset{\sim}{A}^d$, is the n-input n-output logic element whose output functions are $y_1^d,y_2^d, \cdots ,y_n^d$, where y_i^d denotes the dual function of y_i.

Lemma 1: If the n-input n-output logic element $\underset{\sim}{A}$ is conservative, then $\underset{\sim}{A}^d$ is also conservative.

Proof: See Appendix.

It is known that there are $3^6 = 729$ 3-3 CLE's. These CLE's can be classified into 31 equivalence classes under permutations of the inputs and the outputs [7]. For example, the element shown in Fig. 1 belongs the 21st class [7], which is denoted by #21.

Example 1: The dual element of #21 shown in Fig. 1 is #15 shown in Fig. 2.

Definition 3: If $\underset{\sim}{A}$ and its dual $\underset{\sim}{A}^d$ belong to the same equivalence class, then $\underset{\sim}{A}$ is said to be self-dual.

Example 2: #24 shown in Fig. 3 is self-dual. By connecting the crossover circuit as shown in Fig. 4, we can realize a circuit which is equivalent to the dual element of #24.

Definition 4: The dual circuit of $\underset{\sim}{R}$, written $\underset{\sim}{R}^d$, is the circuit which is obtained by replacing every element of $\underset{\sim}{R}$ by its dual element.

By duality, we obtain the following lemma.

Lemma 2: The output functions of $\underset{\sim}{R}^d$ are the dual output functions of $\underset{\sim}{R}$.

Definition 5: The set of logic elements $\boldsymbol{A} = \{\underset{\sim}{A}_1,\underset{\sim}{A}_2, \cdots ,\underset{\sim}{A}_s\}$ is said to have the dual property if $\underset{\sim}{A}_i^d$ belongs to $\boldsymbol{A}$ for $i = 1,2, \cdots ,s$.

By Lemma 2, for circuit $\underset{\sim}{R}$, we can obtain the circuit $\underset{\sim}{R}^d$. So if the set of primitives has the dual property, then it is

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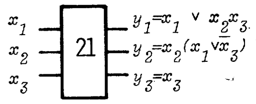


Fig. 1. #21 element.

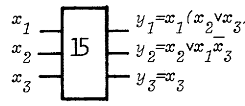


Fig. 2. #15 element.

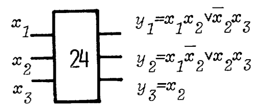


Fig. 3. #24 element.

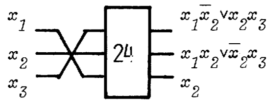


Fig. 4. Circuit which is equivalent to the dual element of #24.

sufficient to consider only 22 equivalence classes shown in Table I out of 31 equivalence classes of 3-3 CLC's.

III. Transform Representation of 3-Input 3-Output CLE's

By the definition of a CLE, the number of "1's" of an input vector is equal to that of the corresponding output vector. In the case of 3-3 CLE, for the input $\mathbf{0} = (0,0,0)$, the output is always $\mathbf{0} = (0,0,0)$ and for the input $\mathbf{1} = (1,1,1)$, the output is always $\mathbf{1} = (1,1,1)$. For the inputs of weight 1, $\boldsymbol{e}_1 = (1,0,0)$, $\boldsymbol{e}_2 = (0,1,0)$, and $\boldsymbol{e}_3 = (0,0,1)$, the outputs are $\boldsymbol{e}_1$ or $\boldsymbol{e}_2$ or $\boldsymbol{e}_3$, respectively. For the inputs of weight 2, $\bar{\boldsymbol{e}}_1 = (0,1,1)$, $\bar{\boldsymbol{e}}_2 = (1,0,1)$, and $\bar{\boldsymbol{e}}_3 = (1,1,0)$, the outputs are $\bar{\boldsymbol{e}}_1$ or $\bar{\boldsymbol{e}}_2$ or $\bar{\boldsymbol{e}}_3$, respectively.

For the inputs $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, $\boldsymbol{e}_3$, $\bar{\boldsymbol{e}}_1$, $\bar{\boldsymbol{e}}_2$, and $\bar{\boldsymbol{e}}_3$, let the outputs be $\boldsymbol{e}_{a_1}$, $\boldsymbol{e}_{a_2}$, $\boldsymbol{e}_{a_3}$, $\bar{\boldsymbol{e}}_{b_1}$, $\bar{\boldsymbol{e}}_{b_2}$, and $\bar{\boldsymbol{e}}_{b_3}$, respectively, where $a_1,a_2,a_3,b_1,b_2,b_3 \in I_3$, $I_3 = \{1,2,3\}$. Thus, the 3-3 CLE μ can be represented as

$$\text{A)}\quad \mu = \frac{(a_1\, a_2\, a_3)}{(b_1\, b_2\, b_3)},$$

where $a_1,a_2,a_3,b_1,b_2,b_3 \in I_3$.

Expression A) is said to be the transform representation of a 3-3 CLE.

Example 3: The transform representations of #15, #21, and #24 are

$$\frac{(2\,2\,3)}{(1\,2\,3)},\ \frac{(1\,2\,3)}{(2\,2\,3)},\ \frac{(2\,3\,1)}{(1\,3\,2)},$$

respectively.

Hereafter, we represent 3-3 CLE's(CLC's) by transform representations.

Lemma 3: If two 3-3 CLE's μ_1 and μ_2 are connected in a cascade, then 3-3 CLE $\mu_1 \cdot \mu_2$ is realized, where

$$\mu_1 = \frac{(a_1\, a_2\, a_3)}{(b_1\, b_2\, b_3)},\quad \mu_2 = \frac{(c_1\, c_2\, c_3)}{(d_1\, d_2\, d_3)},$$

$$\text{and}\quad \mu_1 \cdot \mu_2 = \frac{(a_1\, a_2\, a_3)\cdot(c_1\, c_2\, c_3)}{(b_1\, b_2\, b_3)\cdot(d_1\, d_2\, d_3)}.$$

"$\cdot$" denotes the composition of transformation and $(a_1\, a_2\, a_3)\cdot(c_2\, c_1\, c_3) = (c_{a_1}\, c_{a_2}\, c_{a_3})$.

Example 4: As

$$\frac{(2\,2\,3)}{(1\,2\,3)}\cdot\frac{(1\,2\,3)}{(2\,2\,3)} = \frac{(2\,2\,3)}{(2\,2\,3)},$$

if #15 and #21 are connected in a cascade, then #29 is realized as shown in Fig. 5.

Lemma 4: Let the transform representation of the element $\underline{A}$) be μ. If

$$\mu = \frac{(a_1\, a_2\, a_3)}{(b_1\, b_2\, b_3)},$$

then

$$\mu^d = \frac{(b_1\, b_2\, b_3)}{(a_1\, a_2\, a_3)},$$

where μ^d denotes the transform representation of $\underline{A}^d$.

Definition 6: If μ_1 and μ_2 belong to the same equivalence class under permutations of the inputs and the outputs, then μ_1 and μ_2 are said to be equivalent and written $\mu_1 \equiv \mu_2$.

Lemma 5: Let $(t_1\, t_2\, t_3)$ be an element of S_3 (symmetric group of degree 3). Here we denote $(t_1\, t_2\ \ t_3)$ instead of

$$\begin{pmatrix} 1 & 2 & 3 \\ t_1 & t_2 & t_3 \end{pmatrix}.$$

$$\text{a)}\quad \frac{(a_1\, a_2\, a_3)}{(b_1\, b_2\, b_3)} \equiv \frac{(t_1\, t_2\, t_3)\cdot(a_1\, a_2\, a_3)}{(t_1\, t_2\, t_3)\cdot(b_1\, b_2\, b_3)}$$

$$\text{b)}\quad \frac{(a_1\, a_2\, a_3)}{(b_1\, b_2\, b_3)} \equiv \frac{(a_1\, a_2\, a_3)\cdot(t_1\, t_2\, t_3)}{(b_1\, b_2\, b_3)\cdot(t_1\, t_2\, t_3)}.$$

Premultiplication corresponds to the permutation of columns and post-multiplication corresponds to the permutation of values.

Example 5: As $(2\,1\,3) \in S_3$, we have

$$\text{a)}'\quad \frac{(2\,1\,3)}{(1\,1\,3)} \equiv \frac{(2\,1\,3)\cdot(2\,1\,3)}{(2\,1\,3)\cdot(1\,1\,3)} = \frac{(1\,2\,3)}{(1\,1\,3)},$$

$$\text{b)}'\quad \frac{(1\,2\,3)}{(2\,2\,3)} \equiv \frac{(1\,2\,3)\cdot(2\,1\,3)}{(2\,2\,3)\cdot(2\,1\,3)} = \frac{(2\,1\,3)}{(1\,1\,3)}.$$

From a)$'$ and b)$'$, we obtain

$$\frac{(1\,2\,3)}{(1\,1\,3)} \equiv \frac{(1\,2\,3)}{(2\,2\,3)}.$$

TABLE I
MINIMAL 3-3 CLC'S

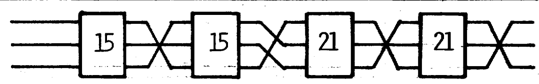
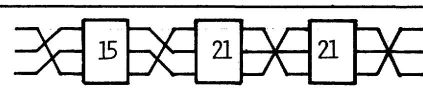
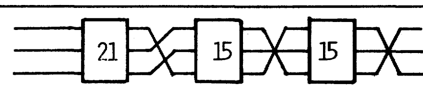
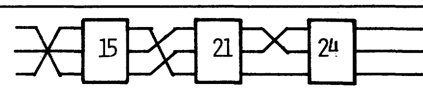
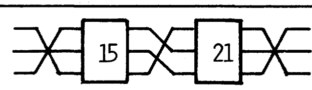
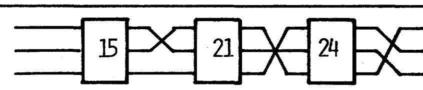
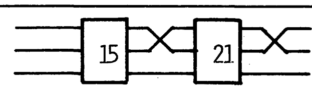
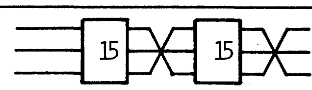
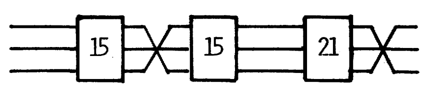
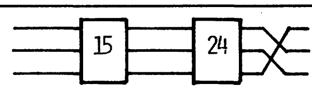
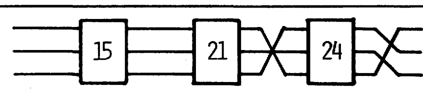
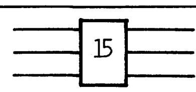
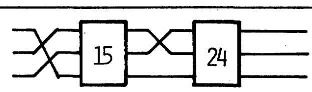
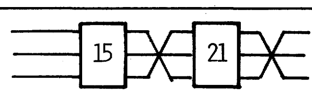
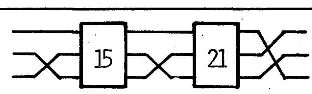
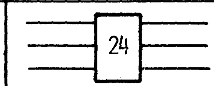
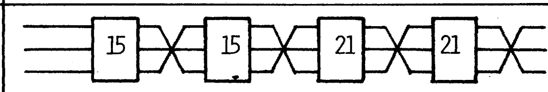
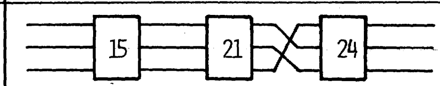
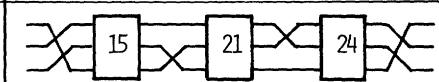
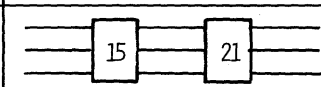
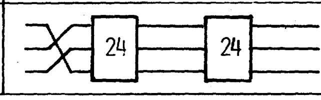
A	A^d	μ	Boolean expression	$\phi_1\phi_2\psi$	Minimal circuit
1	1	$\frac{(3\ 3\ 3)}{(2\ 2\ 2)}$	$x_1x_2 \vee x_2x_3 \vee x_3x_1$ $x_1x_2x_3$ $x_1 \vee x_2 \vee x_3$	3 3 0	
2	3	$\frac{(1\ 3\ 3)}{(2\ 2\ 2)}$	$x_1 \vee x_2x_3$ $x_1x_2x_3$ $x_2 \vee x_3$	1 3 0	
4	18	$\frac{(2\ 2\ 2)}{(1\ 1\ 2)}$	x_1x_2 $x_3 \vee (x_1 \oplus x_2)$ $x_1x_2 \vee x_2x_3 \vee x_3x_1$	3 1 1	
5	5	$\frac{(2\ 3\ 3)}{(1\ 1\ 2)}$	x_1x_2 $x_1\bar{x}_2 \vee x_2x_3$ $x_2 \vee x_3$	1 1 0	
6	14	$\frac{(3\ 1\ 1)}{(2\ 1\ 2)}$	$x_2 \vee \bar{x}_1x_3$ x_1x_3 $x_1 \vee x_2x_3$	1 1 1	
7	11	$\frac{(2\ 2\ 3)}{(1\ 1\ 2)}$	x_1x_2 $x_1x_3 \vee (x_1 \oplus x_2)$ $x_3 \vee x_1x_2$	1 1 0	
8	8	$\frac{(2\ 2\ 3)}{(1\ 1\ 3)}$	x_1x_2 $x_1 \vee x_2$ x_3	1 1 1	
9	19	$\frac{(2\ 2\ 2)}{(1\ 2\ 3)}$	$x_1(x_2 \vee x_3)$ $x_2 \vee (x_1 \oplus x_3)$ $x_3(x_1 \vee x_2)$	3 0 1	
10	28	$\frac{(2\ 2\ 2)}{(1\ 2\ 2)}$	$x_1(x_2 \vee x_3)$ $\bar{x}_1(x_2 \vee x_3) \vee x_1(x_2 \oplus x_3)$ $x_1x_2 \vee x_2x_3 \vee x_3x_1$	3 1 2	
12	20	$\frac{(1\ 1\ 2)}{(2\ 1\ 3)}$	$x_2 \vee x_1x_3$ $x_1x_2 \vee x_2x_3$ $x_3(x_1 \vee x_2)$	1 0 1	
13	13	$\frac{(1\ 1\ 3)}{(1\ 1\ 2)}$	$x_1x_2 \vee x_2x_3 \vee x_3x_1$ $x_3(x_1 \vee x_2)$ $x_3 \vee x_1x_2$	1 1 2	
15	21	$\frac{(2\ 2\ 3)}{(1\ 2\ 3)}$	$x_1(x_2 \vee x_3)$ $x_2 \vee x_1x_3$ x_3	1 0 2	
16	30	$\frac{(1\ 2\ 2)}{(2\ 3\ 1)}$	$x_1\bar{x}_2 \vee x_2x_3$ $x_1x_2 \vee (x_2 \oplus x_3)$ $x_2(x_1 \vee x_3)$	1 0 0	
17	17	$\frac{(2\ 2\ 3)}{(1\ 2\ 2)}$	$x_1(x_2 \vee x_3)$ $x_2x_3 \vee \bar{x}_3(x_1 \oplus x_2)$ $x_3 \vee x_1x_2$	1 1 1	
22	22	$\frac{(2\ 1\ 2)}{(1\ 1\ 2)}$	$x_2(x_1 \vee x_3)$ $x_3 \vee x_1\bar{x}_2$ $x_1x_2 \vee x_2x_3' \vee x_3x_1$	1 1 2	
23	23	$\frac{(1\ 2\ 3)}{(1\ 2\ 3)}$	x_1 x_2 x_3	0 0 3	

TABLE I (CONTINUED)

A	A^d	μ	Boolean expression	$\phi_1\phi_2\psi$	Minimal circuit
24	24	$\frac{(2\ 3\ 1)}{(1\ 3\ 2)}$	$x_1x_2 \vee \bar{x}_2x_3$ $x_1\bar{x}_2 \vee x_2x_3$ x_3	0 0 1	
25	25	$\frac{(2\ 2\ 2)}{(2\ 2\ 2)}$	$x_1x_2 \vee x_2x_3 \vee x_3x_1$ $x_1 \oplus x_2 \oplus x_3$ $x_1x_2 \vee x_2x_3 \vee x_3x_1$	3 3 3	
26	26	$\frac{(1\ 1\ 2)}{(2\ 2\ 1)}$	$x_1x_3 \vee (x_1 \oplus x_2)$ $x_1x_2 \vee \bar{x}_1\bar{x}_2x_3$ $x_1x_2 \vee x_2x_3 \vee x_3x_1$	1 1 0	
27	27	$\frac{(3\ 2\ 2)}{(2\ 2\ 3)}$	$x_1x_2 \vee x_2x_3 \vee x_3x_1$ $x_1x_2 \vee \bar{x}_1(x_2 \oplus x_3)$ $x_1\bar{x}_2 \vee x_2x_3$	1 1 1	
29	29	$\frac{(2\ 2\ 3)}{(2\ 2\ 3)}$	$x_1x_2 \vee x_2x_3 \vee x_3x_1$ $x_1x_2 \vee x_2x_3 \vee x_3x_1$ x_3	1 1 3	
31	31	$\frac{(2\ 3\ 1)}{(3\ 1\ 2)}$	$x_1x_2 \vee x_1x_3$ $x_1x_2 \vee x_2x_3$ $x_1x_3 \vee x_2x_3$	0 0 0	

The first column represents the equivalence class of the element.

The second column represents the equivalence class of the dual element.

The third column represents the transform representation of the element .

The fifth column represents the values of the characteristic functions.

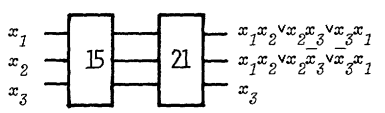


Fig. 5. Realization of #29.

IV. Universal Set of Primitives

In this section, we consider the sets of primitives which are able to realize any 3-3 CLC in a cascade connection.

A. Closed set of CLC's

The set of all the 3-3 CLC's is denoted by K, that is,

$$K = \left\{ \mu = \frac{(a_1\, a_2\, a_3)}{(b_1\, b_2\, b_3)} \;\middle|\; \begin{matrix} a_i, b_j \in I_3 \\ i,j = 1,2,3 \end{matrix} \right\}.$$

Definition 7: Let $M = \{\mu_1, \mu_2, \cdots, \mu_p\}$ be a subset of K. The minimal set S which satisfies the following conditions is said to be the set of composed functions, written $[M]$ or $[\mu_1, \mu_2, \cdots, \mu_p]$: a) $M \subset S$, b) $\mu_i, \mu_j \in S \Rightarrow \mu_i \cdot \mu_j,\ \mu_j \cdot \mu_i \in S$.

$[\mu_1, \mu_2, \cdots, \mu_p]$ represents the set of the circuits which are obtained by cascade connections of $\mu_1, \mu_2, \cdots$, and μ_p. It is clear that $[M] \subset K$ by definition. If $[M] = K$, then M is said to be a universal set of 3-3 CLC's.

Next, we define three characteristic functions ϕ_1, ϕ_2, and ψ. These functions perform an important role in this paper.

Definition 8:

$$\phi_1(\mu) = \delta(a_1 - a_2) + \delta(a_2 - a_3) + \delta(a_3 - a_1)$$

$$\phi_2(\mu) = \delta(b_1 - b_2) + \delta(b_2 - b_3) + \delta(b_3 - b_1)$$

$$\psi(\mu) = \delta(a_1 - b_1) + \delta(a_2 - b_2) + \delta(a_3 - b_3),$$

where

$$\mu = \frac{(a_1\, a_2\, a_3)}{(b_1\, b_2\, b_3)} \in K$$

and

$$\delta(x) = \begin{cases} 0, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0. \end{cases}$$

By Definition 8, it is clear that $\phi_1(\mu)$, $\phi_2(\mu)$, and $\psi(\mu)$ are invariant under permutations of the inputs and outputs of the element.

Lemma 6: Let $\mu_1, \mu_2 \in K$. For $i = 1,2$

a) $\phi_i(\mu_1) = \phi_i(\mu_2) = 0 \Rightarrow \phi_i(\mu_1 \cdot \mu_2) = 0$

b) $\phi_i(\mu_1) = 3 \Rightarrow \phi_i(\mu_1 \cdot \mu_2) = \phi_i(\mu_2 \cdot \mu_1) = 3$

c) $\phi_i(\mu_1) \neq 0 \Rightarrow \phi_i(\mu_1 \cdot \mu_2) \neq 0, \phi_i(\mu_2 \cdot \mu_1) \neq 0.$

d) $\phi_i(\mu_1) = 0, \phi_i(\mu_2) = 1 \Rightarrow \phi_i(\mu_1 \cdot \mu_2) = \phi_i(\mu_2 \cdot \mu_1) = 1.$

Proof: See Appendix.

Lemma 7: Let $\mu_1, \mu_2 \in K$. If $\psi(\mu_1) = \psi(\mu_2) = 3$, then $\psi(\mu_1 \cdot \mu_2) = \psi(\mu_2 \cdot \mu_1) = 3$.

Proof: See Appendix.

Definition 9:

$$M_i = \{\mu \mid \mu \in K, \phi_i(\mu) = 0\} \qquad (i = 1,2)$$
$$L_i = \{\mu \mid \mu \in K, \phi_i(\mu) = 3\} \qquad (i = 1,2)$$
$$N_i = \{\mu \mid \mu \in K, \psi(\mu) = i\} \qquad (i = 1,2,3).$$

By Lemmas 6 and 7, Lemmas 8 and 9 are easily proved.

Lemma 8[1]*:*

$$[M_i] = M_i,\ [L_i] = L_i \qquad (i = 1,2)$$
$$[\overline{M}_i] = \overline{M}_i \qquad (i = 1,2)$$
$$[N_3] = N_3.$$

Lemma 9:

$$[M_1 \cup \overline{M}_2 \cup L_1] = M_1 \cup \overline{M}_2 \cup L_1.$$
$$[\overline{M}_1 \cup M_2 \cup L_2] = \overline{M}_1 \cup M_2 \cup L_2.$$
$$[\overline{M}_1 \cup \overline{M}_2] = \overline{M}_1 \cup \overline{M}_2.$$

Furthermore, we can prove the next lemma.

Lemma 10:

$$[(N_0 \cup N_3) \cap M_1 \cap M_2] = (N_0 \cup N_3) \cap M_1 \cap M_2.$$

Proof: See Appendix.

Now, we can prove that the set of primitives which is able to realize any CLC contains at least three different primitives.

Theorem 1: If M is universal, then M contains three different elements μ_1, μ_2, and μ_3 such that $\mu_1 \in \overline{M}_1 \cap M_2 \cap L_1$, $\mu_2 \in M_1 \cap \overline{M}_2 \cap L_2$, and $\mu_3 \in M_1 \cap M_2 \cap N_1$.

Proof: By Lemma 9, $M_1 \cup \overline{M}_2 \cup L_1$ and $\overline{M}_1 \cup M_2 \cup L_2$ are closed systems and it is easy to verify that $\overline{M}_1 \cap M_2 \cap \overline{L}_1 \neq \phi$ and $M_1 \cap \overline{M}_2 \cap \overline{L}_2 \neq \phi$. So if $[M] = K$, then $M \not\subseteq (M_1 \cup \overline{M}_2 \cup L_1)$ and $M \not\subseteq (\overline{M}_1 \cup M_2 \cup L_2)$. This implies the existence of $\mu_1 \in \overline{M}_1 \cap M_2 \cap L_1$ and $\mu_2 \in M_1 \cap \overline{M}_2 \cap L_2$.

By Lemma 9, $\overline{M}_1 \cup \overline{M}_2$ is a closed system and it is easy to verify that $M_1 \cap M_2 \neq \phi$, so if $[M] = K$, then $M \not\subseteq (\overline{M}_1 \cup \overline{M}_2)$. This implies the existence of $\mu_3 \in M_1 \cap M_2$. Furthermore, by Lemma 10, $(N_0 \cup N_3) \cap M_1 \cap M_2$ is a closed system and it is easy to verify that $M_1 \cap M_2 \cap N_1 \neq \phi$ and $M_1 \cap M_2 \cap N_2 = \phi$, so $\mu_3 \in M_1 \cap M_2 \cap N_1$. Q.E.D.

The fifth column of Table I denotes the values of $\phi_1(\mu)$, $\phi_2(\mu)$, and $\psi(\mu)$ for each equivalence class. By Table I and Theorem 1, we obtain the following corollary.

Corollary 1: If M is universal, then M contains three different elements μ_1, μ_2, and μ_3 such that μ_1 is either #12 or #15 or #16 and μ_2 is either #20 or #21 or #30, and μ_3 is #24.

B. In the Case When Crossovers of Lines are Permitted

Here, we show that any 3-3 CLC can be realized as a cascade connection of three different primitives if the crossovers of lines are permitted.

[1] $\overline{M}_i$ denotes the complement set of M_i, i.e., $K - M_i$.

As to the decomposition of the transformation, the following lemma is known [12].

Lemma 11[2]*:* Any transformation $T = (t_1\ t_2\ t_3)$ can be represented as a composition of three different generations P, C, and D, where $P = (2\ 1\ 3)$, $C = (2\ 3\ 1)$, and $D = (1\ 1\ 3)$.

Lemma 12[3]*:* Any 3-3 CLC can be realized as a cascade connection of six different primitives P/I, C/I, D/I, I/P, I/C, and I/D, where $I = (1\ 2\ 3)$.

Proof: Any 3-3 CLC T_1/T_2 can be realized as a cascade connection of T_1/I and I/T_2. By Lemma 11, T_1/I can be realized as a cascade connection of P/I, C/I, and D/I. Similarly, I/T_2 can be realized as a cascade connection of I/P, I/C, and I/D. Hence, we obtain the lemma. Q.E.D.

Lemma 13: Any 3-3 CLC can be realized as a cascade connection of three different primitives μ_1, μ_2, and μ_3, if the crossovers of lines are permitted, where

$$\mu_1 = \frac{(2\ 2\ 3)}{(1\ 2\ 3)}\ (\#15), \quad \mu_2 = \frac{(1\ 2\ 3)}{(2\ 2\ 3)}\ (\#21),$$
$$\text{and } \mu_3 = \frac{(2\ 3\ 1)}{(1\ 3\ 2)}\ (\#24).$$

Proof: As the crossovers of lines are permitted, elements belonging to the same equivalence class can be regarded as the same elements. Note that

$$\frac{I}{D} \equiv \frac{(1\ 2\ 3)}{(2\ 2\ 3)}\ (\#21), \quad \frac{D}{I} \equiv \frac{(2\ 2\ 3)}{(1\ 2\ 3)}\ (\#15),$$
$$\text{and } \frac{I}{P} \equiv \frac{P}{I} \equiv \frac{(2\ 3\ 1)}{(1\ 3\ 2)}\ (\#24).$$

I/C and C/I can be realized by using two I/P as follows:

$$\frac{I}{P} \cdot \frac{(1\ 3\ 2)}{(1\ 3\ 2)} \cdot \frac{I}{P} \cdot \frac{(1\ 3\ 2)}{(1\ 3\ 2)} \equiv \frac{I}{C} \equiv \frac{C}{I}.$$

Thus any 3-3 CLC can be realized as a cascade connection of three different primitives {#15, #21, #24}. Q.E.D.

Theorem 2: If the crossovers of lines are permitted, then any 3-3 CLE can be realized as a cascade of three different primitives μ_1, μ_2, and μ_3, where $\mu_1 \in \overline{M}_1 \cap M_2 \cap \overline{M}_3$, $\mu_2 \in M_1 \cap \overline{M}_2 \cap \overline{L}_2$, and $\mu_3 \in M_1 \cap M_2 \cap N_1$.

Proof: By Corollary 1, μ_1 is either #12 or #15 or #16, and μ_2 is either #20 or #21 or #30, and μ_3 is #24. By Lemma 13, any 3-3 CLE can be realized as a cascade connection of three different primitives {#15, #21, #24}. Therefore, it is sufficient to show that the set of primitives which contains #12 or #16 instead of #15, and/or #20 or #30 instead of #21 has the same ability as {#15, #21, #24}.

#15 can be realized by #12 and #24 as follows:

[2] If we regard T as a three-valued one-variable logic function, then it can be said that any three-valued one-variable logic function is realizable by the composition of three different primitive three-valued one-variable logic functions.

[3] This lemma implies that the method of realization of 3-3 CLC's is similar to that of a pair of three-valued one-variable logic functions.

$$\frac{(2\,2\,3)}{(1\,2\,3)} = \frac{(1\,1\,2)}{(2\,1\,3)} \cdot \frac{(2\,1\,3)}{(2\,1\,3)} \cdot \frac{(2\,3\,1)}{(1\,3\,2)} \cdot \frac{(1\,3\,2)}{(1\,3\,2)}.$$

Similarly, #15 can be realized by #16 and #24 as follows:

$$\frac{(2\,2\,3)}{(1\,2\,3)} = \frac{(2\,3\,1)}{(2\,3\,1)} \cdot \frac{(1\,2\,2)}{(2\,3\,1)} \cdot \frac{(1\,3\,2)}{(1\,3\,2)} \cdot \frac{(2\,3\,1)}{(1\,3\,2)} \cdot \frac{(2\,3\,1)}{(2\,3\,1)}.$$

By duality, it is easy to observe that #21 can be realized by #20 and #24, or by #30 and #24. Hence, any 3-3 CLC can be realized as a cascade connection of $\{\mu_1,\mu_2,\mu_3\}$. Q.E.D.

C. *In the Case When the Crossovers of Lines are not Permitted*

Here, we show that the necessary and sufficient number of different primitives to realize an arbitrary 3-3 CLC as a cascade connection is four, if the crossovers of lines are not permitted.

Theorem 3: If M is universal, then M contains at least four elements.

Proof: Let $[\mu_1,\mu_2,\cdots,\mu_p] = K$. By Theorem 1, it is clear that $p \geq 3$, and let $\mu_1 \in \overline{M}_1 \cap M_2 \cap \overline{L}_1$, $\mu_2 \in M_1 \cap \overline{M}_2 \cap \overline{L}_2$ and $\mu_3 \in M_1 \cap M_2 \cap N_1$. As $\mu_1 \in \overline{M}_1$ and $\mu_2 \in \overline{M}_2$, $\mu_1\mu_j,\mu_j\mu_1 \in \overline{M}_1$ and $\mu_2 \cdot \mu_j, \mu_j \cdot \mu_2 \in \overline{M}_2$ $(j = 1,2,\cdots,p)$, by Lemma 6. Therefore if $p = 3$, then $[\mu_3] = M_1 \cap M_2$. On the other hand, $M_1 \cap M_2$ is isomorphic to $S_3 \times S_3$ because

$$M_1 \cap M_2 = \left\{\mu = \frac{(a_1\,a_2\,a_3)}{(b_1\,b_2\,b_3)} \;\middle|\; \begin{matrix}(a_1\,a_2\,a_3) \in S_3\\ (b_1\,b_2\,b_3) \in S_3\end{matrix}\right\}.$$

$S_3 \times S_3$ is not a cyclic group, so $[\mu_3] \neq M_1 \cap M_2$. Hence $p \geq 4$. Q.E.D.

Next, we show that any 3-3 CLE which belongs to $M_1 \cap M_2$ can be realized by a cascade connection of two different primitives.

Lemma 14 [12]: An arbitrary element of S_3 can be represented as a composition of $P = (2\,1\,3)$ and $C = (2\,3\,1)$.

Lemma 15: $[\mu_3,\mu_4] = M_1 \cap M_2$, where

$$\mu_3 = \frac{(2\,3\,1)}{(1\,3\,2)} \quad \text{and } \mu_4 = \frac{(2\,1\,3)}{(2\,1\,3)}.$$

Proof: Note that

$$(\mu_3)^4 = \frac{C}{I}, \quad (\mu_4 \cdot \mu_3)^3 \cdot (\mu_3)^2 = \frac{P}{I}$$

$$(\mu_4 \cdot \mu_3)^2 = \frac{I}{C}, \quad (\mu_3)^3 \cdot (\mu_4 \cdot \mu_3)^2 = \frac{I}{P}.$$

From Lemma 14, we obtain the lemma. Q.E.D.

It should be noted that any crossover is contained in $M_1 \cap M_2$.

Theorem 4: $\{\mu_1,\mu_2,\mu_3,\mu_4\}$ is universal, where

$$\mu_1 = \frac{(2\,2\,3)}{(1\,2\,3)},\ \mu_2 = \frac{(1\,2\,3)}{(2\,2\,3)},\ \mu_3 = \frac{(2\,3\,1)}{(1\,3\,2)},\text{ and } \mu_4 = \frac{(2\,1\,3)}{(2\,1\,3)}.$$

By Theorems 3 and 4, the necessary and sufficient number of different primitives to realize an arbitrary 3-3 CLC as a cascade connection is four if the crossover of lines are not permitted.

Proof: By Theorem 2 and Lemma 15. Q.E.D.

V. Minimal 3-Input 3-Output Conservative Logic Circuits

In this section, we consider the realization of 3-3 CLC's with minimum numbers of primitives in the case when crossovers of the lines are permitted.

As stated in Section II, if the set of primitives have the dual property, then it is easy to realize the circuit, so we use {#15, #21, #24} as the set of primitives.

As shown in the proof of Theorem 2, the synthesis problem of the circuit μ can be reduced to the problem of decomposing μ into the following transformations:

1) Transformations Which Correspond to the Primitives:

$$\frac{(2\,2\,3)}{(1\,2\,3)},\ \frac{(1\,2\,3)}{(2\,2\,3)},\text{ and } \frac{(2\,3\,1)}{(1\,3\,2)}.$$

2) Transformations Which Correspond to the Crossovers of Lines:

$$\frac{(1\,3\,2)}{(1\,3\,2)},\ \frac{(3\,2\,1)}{(3\,2\,1)},\ \frac{(2\,1\,3)}{(2\,1\,3)},\ \frac{(2\,3\,1)}{(2\,3\,1)},\text{ and } \frac{(3\,1\,2)}{(3\,1\,2)}.$$

As the crossovers of lines can be done freely, the decomposition containing the minimum number of transformations which correspond to the primitives must be found.

The following example illustrates the method used here.

Example 6: #10 can be decomposed into two circuits:

$$\frac{(2\,2\,2)}{(1\,2\,2)} = \frac{(2\,2\,2)}{(1\,2\,3)} \cdot \frac{(1\,2\,3)}{(1\,2\,2)}. \tag{1}$$

So we must decompose $\frac{(2\,2\,2)}{(1\,2\,3)}$ and $\frac{(1\,2\,3)}{(1\,2\,2)}$. Note that

$$\frac{(2\,2\,2)}{(1\,2\,3)} = \frac{(2\,2\,3)}{(1\,2\,3)} \cdot \frac{(1\,2\,2)}{(1\,2\,3)}. \tag{2}$$

and

$$\frac{(1\,2\,2)}{(1\,2\,3)} = \frac{(3\,2\,1)}{(3\,2\,1)} \cdot \frac{(2\,2\,3)}{(1\,2\,3)} \cdot \frac{(3\,2\,1)}{(3\,2\,1)}. \tag{3}$$

Consider the dual of (3),

$$\frac{(1\,2\,3)}{(1\,2\,2)} = \frac{(3\,2\,1)}{(3\,2\,1)} \cdot \frac{(1\,2\,3)}{(2\,2\,3)} \cdot \frac{(3\,2\,1)}{(3\,2\,1)}. \tag{4}$$

From (1) to (4), we obtain

$$\frac{(2\,2\,2)}{(1\,2\,2)} = \frac{(2\,2\,3)}{(1\,2\,3)} \cdot \frac{(3\,2\,1)}{(3\,2\,1)} \cdot \frac{(2\,2\,3)}{(1\,2\,3)} \cdot \frac{(1\,2\,3)}{(2\,2\,3)} \cdot \frac{(3\,2\,1)}{(3\,2\,1)}. \tag{5}$$

The decomposition of (5) corresponds to the circuit of #10 shown in Table I. (The End of Example 6.)

The other circuits can be realized in a similar way to

Example 6. In general, for a 3-3 CLC, the decomposition which contains minimum number of primitives is not unique. In Table I, the circuits which contain minimum number of #15 and #21 are shown. The necessary and sufficient number of #15 and #21 to realize a circuit can be obtained by the following theorem.

Theorem 5: When the crossovers of lines are permitted, any circuit is realized as a cascade connection of three different primitives $\{\mu_1,\mu_2,\mu_3\}$. The necessary and sufficient number of μ_i are given by $\lambda_i(\mu)$, where $\mu_1 \in \overline{M}_1 \cap M_2 \cap \overline{L}_1$, $\mu_2 \in M_1 \cap \overline{M}_2 \cap \overline{L}_2$, $\mu_3 \in M_1 \cap M_2 \cap N_1$ and $\lambda_i(\mu) = \lceil \frac{1}{2} \cdot \phi_i(\mu) \rceil (i = 1,2)$.

$\lceil a \rceil$ denotes the minimum integer not less than a.

Proof: If $\phi_1(\mu) \neq 0$, i.e., $\mu \in \overline{M}_1$, then at least one μ_1 is necessary because $[\mu_2,\mu_3] \subseteq M_1$. If $\phi_1(\mu) = 3$, then at least two μ_1 is necessary because $\phi_1(\mu_1) = 1$ and by d) of Lemma 6. For μ_2, the similar argument can be done.

By Table I and the proof of Theorem 2, it is easy to show the sufficiency. Q.E.D.

Example 7: The minimal circuit of #10 in Example 6 contains at least two #15 and one #21, because

$$\lambda_1(\mu) = \left\lceil \frac{1}{2}\{\delta(2-2) + \delta(2-2) + \delta(2-2)\} \right\rceil = 2$$

$$\lambda_2(\mu) = \left\lceil \frac{1}{2}\{\delta(1-2) + \delta(2-2) + \delta(2-1)\} \right\rceil = 1.$$

So the circuit shown in Table I is minimal.

VI. Conclusion and Comments

In this paper, we considered the problem to realize 3-3 CLC's by cascade connections of 3-3 CLE's called primitives. It is shown that the necessary and sufficient number of different primitives to realize an arbitrary 3-3 CLC is three in the case when the crossovers of lines are permitted, and four in thecase when the crossovers of lines are not permitted.

Appendix

Proof of Lemma 1: As $\underline{A}$ is conservative, for any input vector $\mathbf{a} = (a_1,a_2,\cdots,a_n) \in B^n$,

$$\sum_{i=1}^{n} a_i = \sum_{j=1}^{n} y_j(\mathbf{a}). \tag{A1}$$

By the definition of the dual function,

$$y_j^d(\mathbf{a}) = \overline{y_j(\overline{\mathbf{a}})}$$

and we have

$$\sum_{j=1}^{n} y_j^d(\mathbf{a}) = \sum_{j=1}^{n} (1 - y_j(\overline{\mathbf{a}})) = n - \sum_{j=1}^{n} y_j(\overline{\mathbf{a}}) \tag{A2}$$

By (A1),

$$\sum_{j=1}^{n} y_j(\overline{\mathbf{a}}) = \sum_{i=1}^{n} \overline{a}_i = \sum_{i=1}^{n} (1 - a_i) = n - \sum_{i=1}^{n} a_i. \tag{A3}$$

By (A2) and (A3), we have

$$\sum_{i=1}^{n} a_i = \sum_{j=1}^{n} y_j^d(\mathbf{a}).$$

This implies that $\underline{A}^d$ is also conservative. Q.E.D.

Proof of Lemma 6: Let

$$\mu_1 = \frac{(a_1\,a_2\,a_3)}{(b_1\,b_2\,b_3)}, \quad \mu_2 = \frac{(c_1\,c_2\,c_3)}{(d_1\,d_2\,d_3)}.$$

1) If $\phi_1(\mu_1) = \phi_1(\mu_2) = 0$, then $a_1 \neq a_2$, $a_2 \neq a_3$, $a_3 \neq a_1$, $c_1 \neq c_2$, $c_2 \neq c_3$, and $c_3 \neq c_1$. This implies $(a_1\,a_2\,a_3) \in S_3$ and $(c_1\,c_2\,c_3) \in S_3$. S_3 is closed under the composition, and we have $(a_1\,a_2\,a_3) \cdot (c_1\,c_2\,c_3) = (c_{a_1}\,c_{a_2}\,c_{a_3}) \in S_3$. This implies $c_{a_1} \neq c_{a_2}$, $c_{a_2} \neq c_{a_3}$, and $c_{a_3} \neq c_{a_1}$. Similarly, $(c_1\,c_2\,c_3) \cdot (a_1\,a_2\,a_3) = (a_{c_1}\,a_{c_2}\,a_{c_3}) \in S_3$. This implies $a_{c_1} \neq a_{c_2}$, $a_{c_2} \neq a_{c_3}$, and $a_{c_3} \neq a_{c_1}$. Therefore $\phi_1(\mu_1 \cdot \mu_2) = \phi_1(\mu_2 \cdot \mu_1) = 0$.

2) If $\phi_1(\mu_1) = 3$, then $a_1 = a_2 = a_3$. And we have

$$\mu_1 \cdot \mu_2 = \frac{(a_1\,a_2\,a_3) \cdot (c_1\,c_2\,c_3)}{(b_1\,b_2\,b_3) \cdot (d_1\,d_2\,d_3)} = \frac{(c_{a_1}\,c_{a_1}\,c_{a_1})}{(b_1\,b_2\,b_3) \cdot (d_1\,d_2\,d_3)}$$

and

$$\mu_2 \cdot \mu_1 = \frac{(c_1\,c_2\,c_3) \cdot (a_1\,a_2\,a_3)}{(d_1\,d_2\,d_3) \cdot (b_1\,b_2\,b_3)} = \frac{(a_1\,a_1\,a_1)}{(d_1\,d_2\,d_3) \cdot (b_1\,b_2\,b_3)}.$$

Therefore $\phi_1(\mu_1 \cdot \mu_2) = \phi_1(\mu_2 \cdot \mu_1) = 3$.

3) If $\phi_1(\mu_1) \neq 0$, then $a_1 = a_2$, $a_2 = a_3$, or $a_3 = a_1$. This implies $c_{a_1} = c_{a_2}$, $c_{a_2} = c_{a_3}$, or $c_{a_3} = c_{a_1}$, and also implies $a_{c_1} = a_{c_2}$, $a_{c_2} = a_{c_3}$, or $a_{c_3} = a_{c_1}$. Therefore $\phi_1(\mu_1 \cdot \mu_2) \neq 0$ and $\phi_1(\mu_2 \cdot \mu_1) \neq 0$.

4) If $\phi_1(\mu_1) = 0$, then $a_1 \neq a_2$, $a_2 \neq a_3$, and $a_3 \neq a_1$. If $\phi_1(\mu_2) = 1$, then exactly one of the following holds: $c_1 = c_2$, $c_2 = c_3$, $c_3 = c_1$. So $\phi_1(\mu_1) = 0$ and $\phi_1(\mu_2) = 1$ implies that exactly one of the following holds: $c_{a_1} = c_{a_2}$, $c_{a_2} = c_{a_3}$, $c_{a_3} = c_{a_1}$. Similarly, exactly one of the following holds: $a_{c_1} = a_{c_2}$, $a_{c_2} = a_{c_3}$, $a_{c_3} = a_{c_1}$. Therefore $\phi_1(\mu_1 \cdot \mu_2) = \phi_1(\mu_2 \cdot \mu_1) = 1$. For ϕ_2, similar arguments can be done. Q.E.D.

Proof of Lemma 7: If $\psi(\mu_1) = \psi(\mu_2) = 3$, then $a_i = b_i$ and $c_i = d_i (i = 1,2,3)$. This implies $c_{a_i} = d_{b_i}$ and $a_{c_1} = b_{d_i} (i = 1,2,3)$. Therefore $\psi(\mu_1 \cdot \mu_2) = (\mu_2 \cdot \mu_1) = 3$.

Proof of Lemma 10: By Lemma 8, $[N_3] = N_3$, $[M_1] = M_1$, and $[M_2] = M_2$. This implies $[N_3 \cap M_1 \cap M_2] = N_3 \cap M_1 \cap M_2$. It is easily verified that $N_2 \cap M_1 \cap M_2 = \phi$, $N_i \cap N_j = \phi$ $(i \neq j)$. Therefore, $(N_0 \cup N_1 \cup N_3) \cap M_1 \cap M_2 = M_1 \cap M_2$.

1) First, we show $\mu_1,\mu_2 \in N_0 \cap M_1 \cap M_2 \Rightarrow \mu_1 \cdot \mu_2 \in (N_0 \cup N_3) \cap M_1 \cap M_2$. Let $\mu_1,\mu_2 \in N_0 \cap M_1 \cap M_2$. It is clear that $(a_1\,a_2\,a_3) \in S_3$, $(b_1\,b_2\,b_3) \in S_3$, $(c_1\,c_2\,c_3) \in S_3$, and $(d_1\,d_2\,d_3) \in S_3$. S_3 is closed under the composition, so we have $(c_{a_1}\,c_{a_2}\,c_{a_3}) \in S_3$ and $(d_{b_1}\,d_{b_2}\,d_{b_3}) \in S_3$. Suppose $c_{a_1} = d_{b_1}$, $c_{a_2} \neq d_{b_2}$, $c_{a_3} \neq d_{b_3}$, we have $c_{a_2} = d_{b_3}$ and $c_{a_3} = d_{b_2}$. As $\mu_2 \in N_0$, $c_i \neq d_i$ $(i = 1,2,3)$. This implies $a_2 \neq b_3$ and $a_3 \neq b_2$. Note that $a_2 \neq b_2$ and $a_3 \neq b_3$ because $\mu_1 \in N_0$. Therefore, we have $a_2 = b_1$ and $a_3 = b_1$. But this contradicts $(a_1\,a_2\,a_3) \in S_3$. So there is no case such that $c_{a_1} = d_{b_1}$, $c_{a_2} \neq d_{b_2}$, and $c_{a_3} \neq d_{b_3}$.

Similarly, we can show that there is no case such that $c_{a_1} \neq d_{b_1}$, $c_{a_2} = d_{b_2}$, and $c_{a_3} \neq d_{b_3}$, nor the case such that $c_{a_1} \neq d_{b_1}$, $c_{a_2} \neq d_{b_2}$, and $c_{a_3} = d_{b_3}$. Note that $N_2 \cap M_1 \cap M_2$

$= \phi$, and we have either $c_{a_i} = d_{b_i}$ $(i = 1,2,3)$ or $c_{a_i} \neq d_{b_i}$ $(i = 1,2,3)$. Hence, $\mu_1 \cdot \mu_2 \in (N_0 \cup N_3) \cap M_1 \cap M_2$.

2) Second, we show $\mu_1 \in N_0 \cap M_1 \cap M_2$, $\mu_2 \in N_3 \cap M_1 \cap M_2 \Rightarrow \mu_1 \cdot \mu_2$, $\mu_2 \cdot \mu_1 \in N_0 \cap M_1 \cap M_2$.

As $\mu_1 \in N_0$, $a_i \neq b_i$ $(i = 1,2,3)$. As $\mu_2 \in M_1$, $(c_1\, c_2\, c_3) \in S_3$. This implies $c_1 \neq c_2$, $c_2 \neq c_3$, and $c_3 \neq c_1$. So we have $c_{a_i} \neq c_{b_i}$ $(i = 1,2,3)$. Note that $c_i = d_i$ $(i = 1,2,3)$ because $\mu_2 \in N_3$, and we have $c_{a_i} \neq d_{b_i}$. Hence, $\mu_1 \cdot \mu_2 \in N_0 \cap M_1 \cap M_2$.

$$a_i \neq b_i \ (i = 1,2,3) \text{ and } c_i = d_i \ (i = 1,2,3) \quad \text{implies } a_{c_i} \neq b_{d_i} \ (i = 1,2,3).$$

Hence, $\mu_2 \cdot \mu_1 \in N_0 \cap M_1 \cap M_2$.

3) Lastly, we show $\mu_1, \mu_2 \in N_3 \cap M_1 \cap M_2 \Rightarrow \mu_1 \cdot \mu_2 \in N_3 \cap M_1 \cap M_2$.

By Lemma 8, $[N_3] = N_3$, $[M_1] = M_1$, and $[M_2] = M_2$. So it is clear that 3) holds. From 1), 2), and 3), we obtain the lemma. Q.E.D.

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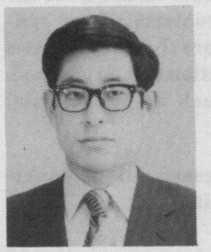

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